AP Calculus AB
Integration By Parts Notes

Name Key

Recall:

$$\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

Therefore:

$$f(x)g(x) = \int f'(x)g(x) + f(x)g'(x)\, dx$$

$$f(x)g(x) = \int f'(x)g(x)\, dx + \int f(x)g'(x)\, dx$$

By rearranging:

$$\int f(x)g'(x)\, dx = f(x)g(x) - \int g(x)f'(x)\, dx$$

Let:

$u = f(x)$

$\frac{du}{dx} = f'(x)$ $\qquad$ $dv = g'(x)\, dx$

$du = f'(x)\, dx$ $\qquad$ $v = g(x)$

Formula for Integration By Parts:

$$\int u\, dv = uv - \int v\, du$$

What to make "u":

L – logarithms

I – inverses

P – polynomials

E – exponentials

T – trig

Ex: $\int x \sin x \, dx$

$u = x$ $\qquad\qquad$ $\int dv = \int \sin x \, dx$

$\frac{du}{dx} = 1$ $\qquad\qquad$ $v = -\cos x$

$du = dx$

$-x\cos x + \int \cos x \, dx$

$-x\cos x + \sin x + C$

Ex: $\int \ln x \, dx$

$u = \ln x$ $\qquad\qquad$ $\int dv = \int dx$

$\frac{du}{dx} = \frac{1}{x}$ $\qquad\qquad$ $v = x$

$du = \frac{1}{x} dx$

$x\ln x - \int x \cdot \frac{1}{x} dx$

$x\ln x - \int 1 \, dx$

$x\ln x - x + C$

Ex: $\int x^2 e^x dx$

$u = x^2$ $\qquad\qquad \int dv = \int e^x dx$

$\frac{du}{dx} = 2x$ $\qquad\qquad v = e^x$

$du = 2x\,dx$

$x^2 e^x - \int 2x e^x dx$

$\boxed{x^2 e^x - 2}\int x e^x dx$

$u = x$ $\qquad\qquad \int dv = \int e^x dx$

$du = dx$ $\qquad\qquad v = e^x$

$x e^x - \int e^x dx$

$x e^x - e^x$

$x^2 e^x - 2x e^x + 2e^x + C$

Ex: $\int e^x \sin x\, dx$

$u = e^x$ $\qquad\qquad \int dv = \int \sin x\, dx$

$\frac{du}{dx} = e^x$ $\qquad\qquad v = -\cos x$

$du = e^x dx$

$\boxed{-e^x \cos x +}\int e^x \cos x\, dx$

$u = e^x$ $\qquad\qquad \int dv = \int \cos x\, dx$

$\frac{du}{dx} = e^x$ $\qquad\qquad v = \sin x$

$du = e^x dx$

$e^x \sin x - \int e^x \sin x\, dx$

$\int e^x \sin x\, dx = -e^x \cos x + e^x \sin x - \int e^x \sin x\, dx$

$2\int e^x \sin x\, dx = -e^x \cos x + e^x \sin x$

$\int e^x \sin x\, dx = \frac{1}{2}(-e^x \cos x + e^x + \sin x) + C$